



Mathematics: analysis and approaches

Standard level

Paper 2

25 October 2024

Zone A morning | Zone B morning | Zone C morning

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

870

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \leq x \leq 20$.

(a) Find the value of

(i) $f(0)$;

(ii) $f(20)$.

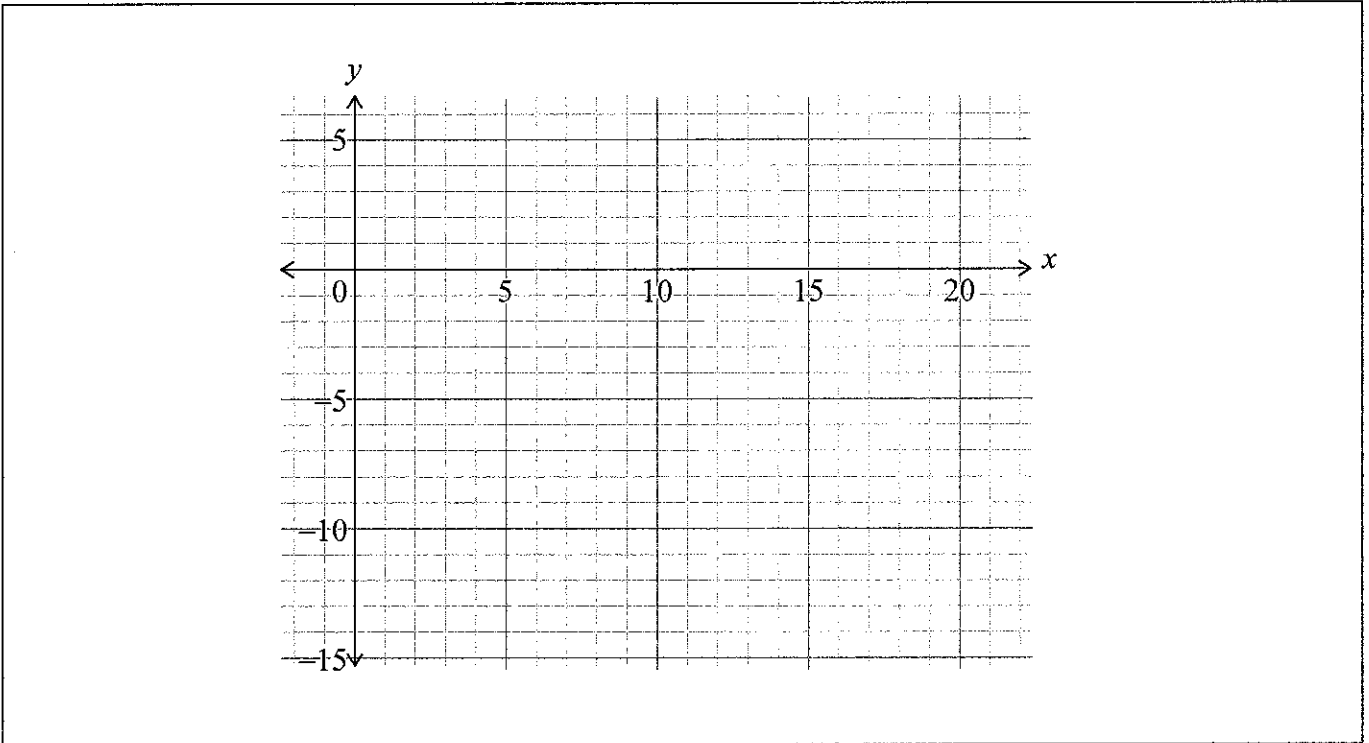
[2]

(b) Find the two roots of $f(x) = 0$.

[2]

(c) Sketch the graph of $y = f(x)$ on the following grid.

[3]



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(Question 1 continued)

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2. [Maximum mark: 4]

Find the coefficient of x^6 in the expansion of $(2x - 5)^9$.

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3. [Maximum mark: 6]

(a) Find $\int (e^{4x} + 6) dx$. [3]

It is given that $h'(x) = e^{4x} + 6$ and $h(1.5) = 105$.

(b) Find $h(x)$. [3]

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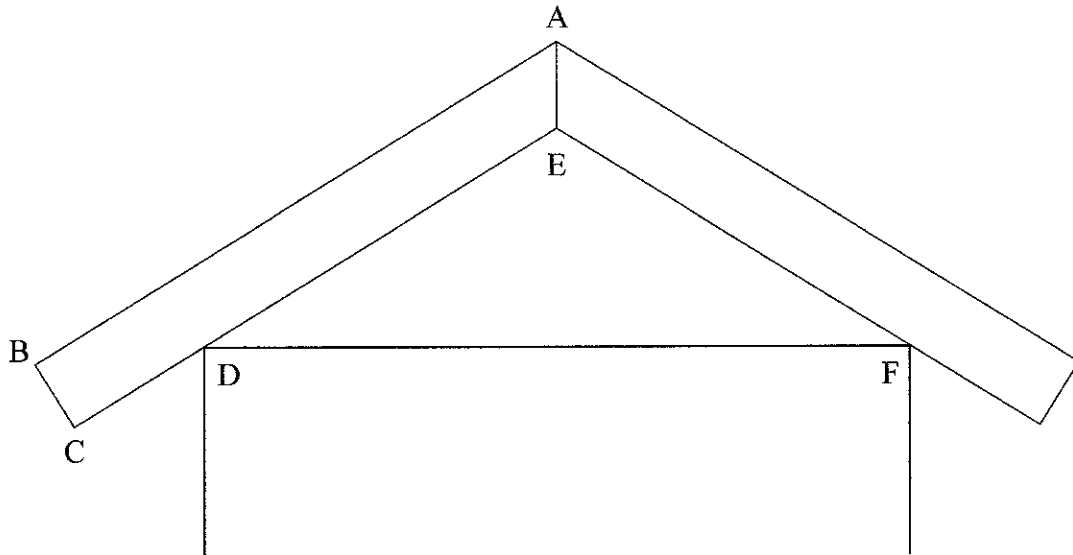
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5. [Maximum mark: 8]

The following diagram shows the cross section of the roof of a house. The cross section is symmetrical about the vertical line through points A and E.

diagram not to scale



The gradient of [BA] is $\frac{7}{12}$.

(a) Find the size of \hat{BAE} , expressing your answer in degrees.

[3]

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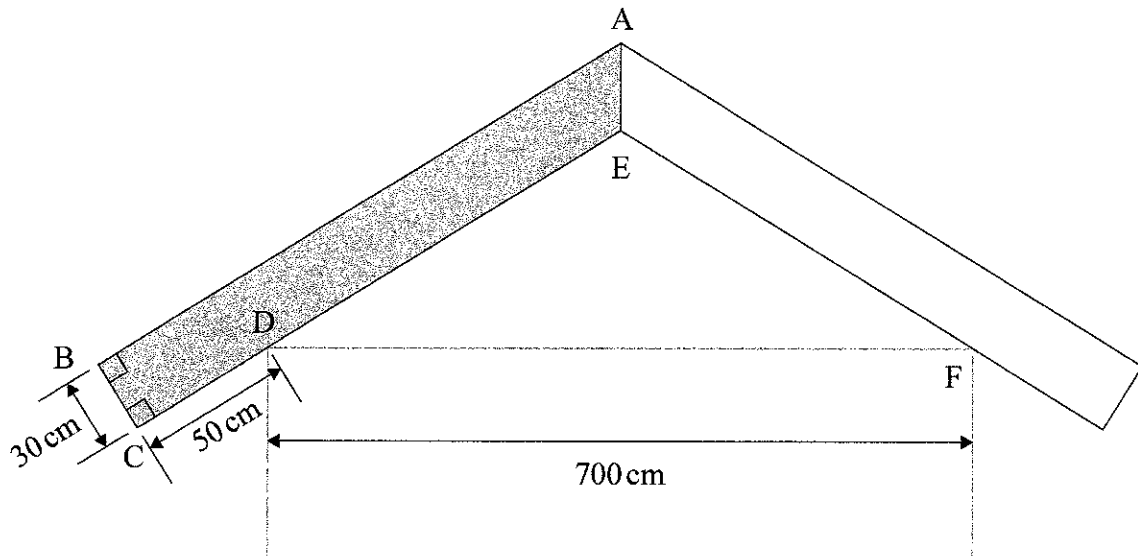
(Question 5 continued)

A builder requires the lengths of the sides [BA] and [CE].

The builder has the following measurements:

$\hat{A}BC = \hat{B}CE = 90^\circ$, $DC = 50$ cm, $BC = 30$ cm, and $DF = 700$ cm.

diagram not to scale



- (b) Find
 - (i) CE;
 - (ii) BA.

[5]

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Turn over

6. [Maximum mark: 5]

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r .

[5]

A large rectangular area with horizontal dotted lines for writing the answer.

Do not write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

MyLife is a social media platform with 93.6 million users, all aged 13 years old and above. The following frequency table shows the number of users by age group.

Age, a (years)	Millions of users
$13 \leq a < 18$	5.5
$18 \leq a < 25$	23.6
$25 \leq a < 45$	43.5
$45 \leq a < 65$	17.3
$65 \leq a < 85$	3.7
Total	93.6

- (a) Find the percentage of *MyLife* users aged 45 years or older. [1]
- (b) A sample of 1000 *MyLife* users is chosen at random. Find the probability that fewer than 200 of them are aged 45 years or older. [3]
- (c) Given that a *MyLife* user chosen at random is 45 years or older, find the probability that they are 65 years or older. [4]
- (d) List the mid-interval value for each class interval. [1]
- (e) Hence, for *MyLife* users, estimate
- (i) the mean age;
- (ii) the variance of the ages. [3]

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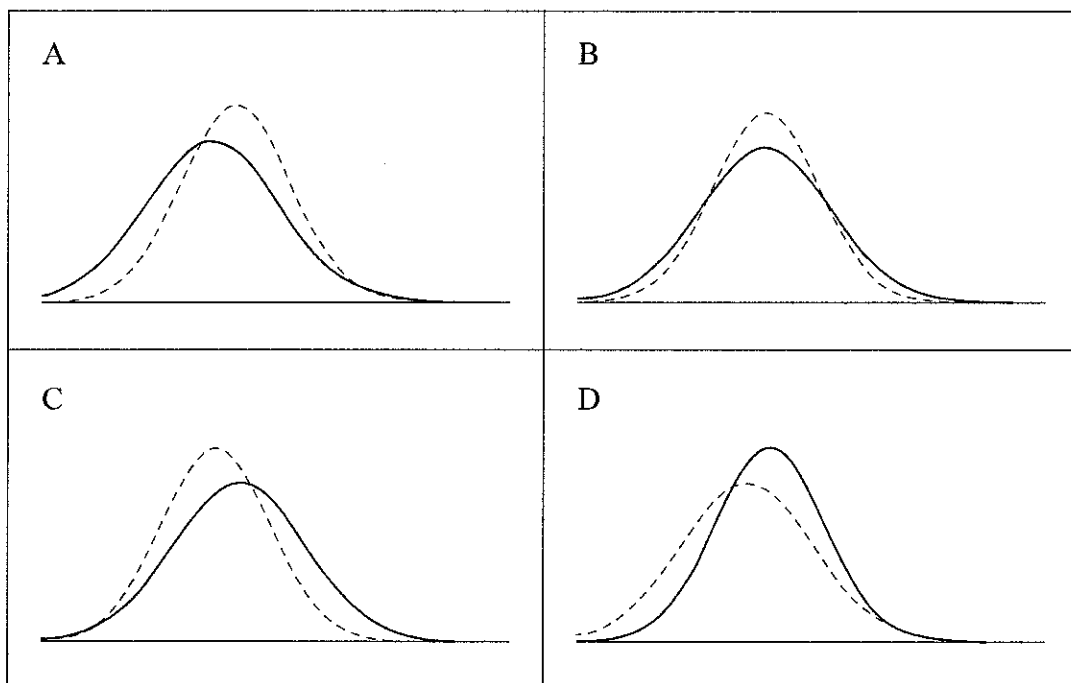
(Question 7 continued)

A different social media platform, *SmallTalk*, reports that its users have a mean age of 29.9 years and a variance of 137 years².

The following four diagrams represent age distributions.

- (f) (i) Identify the diagram which best represents the age distributions for the users of *MyLife* and *SmallTalk*.
- (ii) In your chosen diagram, identify which social media platform is represented by the solid line.

[2]



Do **not** write solutions on this page.

8. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
- (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
- (ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

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(Question 8 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]
- (e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]
- (f) Find the value of
 - (i) $B'(75)$;
 - (ii) $C'(75)$. [2]
- (g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]

Do not write solutions on this page.

9. [Maximum mark: 16]

In this question all values of x and t are in radians.

Consider the function $f(x) = 6 \cos(\pi x)$.

(a) (i) Write down the amplitude of the graph of f .

(ii) Find the period of f .

[3]

Consider a second function $g(x) = -8 \sin(\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a \cos(b(x - c))$, where $a, b, c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(i) the value of a ;

(ii) the value of b ;

(iii) the smallest possible value of c .

[4]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in ms^{-1} , can be modelled by the function $v(t) = -4.5 \cos\left(\frac{\pi}{19}(t - 4)\right) + 10.5$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity.

[1]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion.

[1]

(e) (i) Find $v'(t)$.

(ii) Hence, or otherwise, write down the maximum acceleration of the car.

[4]

(f) Find the distance, in metres, between consecutive speed bumps.

[3]